half-wave rectifier with pi-filter (D_5, C_2, L, and C_3), and a feedback circuit that controls the output voltage by modulating the duty cycle of the switch. The transformer steps down the voltage and provides isolation from the high voltage ac line input. Diode D_6 and R clamp the inductor voltage when the switch opens. The feedback signal path is isolated from the input using an optical isolator. (See Electronics in Action in Chapter 5 for discussion of the optical isolator.)

3.18 dc-to-dc CONVERTERS (ADVANCED TOPIC)

Multiple power supply voltages are required in most electronic systems. One method of generating these voltages is to use several half-wave or full-wave rectifier circuits. However, the output voltage of these circuits is determined by the transformer voltage, and hence multiple transformer outputs are required. In addition, the rectifier circuits most often operate at 60 Hz, and the transformers have a significant size and weight.

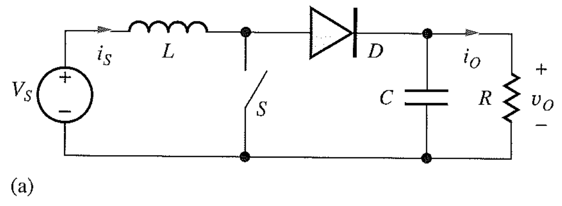
A more flexible method is to use high-efficiency **dc-to-dc converters** that can operate at much higher frequencies, which, as we shall later see, reduces the size and weight of the inductances in the circuit. The dc-to-dc converter uses a dc input voltage and provides an electronically controlled output voltage with a continuously variable range. This section describes two examples of dc-to-dc converters: the **boost converter,** which provides an output voltage that is greater than the input voltage, and the **buck converter,** whose output voltage is less than its input voltage.

3.18.1 The Boost Converter

The circuit for the basic boost converter is in Fig. 3.73(a). At the heart of the converter are the inductor L and switch S, which is periodically turned on and off, as indicated in Fig. 3.73(b). The switch is closed during the time interval T_{on} and open during the time interval T_{off}. Switching is periodic with period $T = T_{on} + T_{off}$. Diode D also operates as a switch and is off when S is on and vice versa. A dc input voltage is supplied by source V_S, and R and C represent the load resistance and filter capacitor, respectively.

In these analyses, we assume that the circuit has been operating for a long time and that any start-up transients have died out. The circuit will then be operating in the periodic steady state.

Switch S Closed During the time interval T_{on}, switch S is closed, and the output voltage, which we will find to be greater than zero, reverse-biases diode D, resulting in the equivalent circuit in Fig. 3.74(a). For simplicity, the ideal model is used for the diode. The dc input voltage V_S now



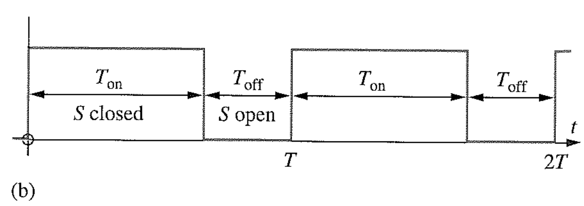


Figure 3.73 (a) dc-to-dc boost converter. (b) Periodic switch timing: Switch S is closed during T_{on} and open during T_{off}.

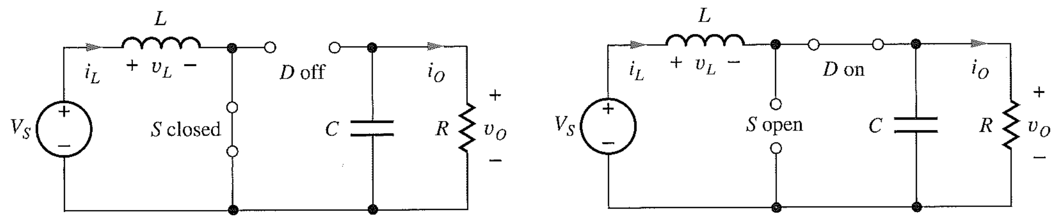


Figure 3.74 (a) Model valid during T_{on} when switch S is closed. (b) Model for T_{off} interval when switch S is open.

appears directly across the inductor, and the inductor current at the end of the T_{on} interval is

$$i_L(T_{on}) = i_L(0^+) + \int_0^{T_{on}} \frac{V_S}{L}\, dt = i_L(0^+) + \frac{V_S}{L} T_{on} \tag{3.79}$$

During time interval $(0, T_{on})$, the inductor current increases at a constant rate, as depicted in Fig. 3.75. Because the current in the inductor cannot change instantaneously, $i_L(0^+)$ is equal to the current just before the switch changes state.

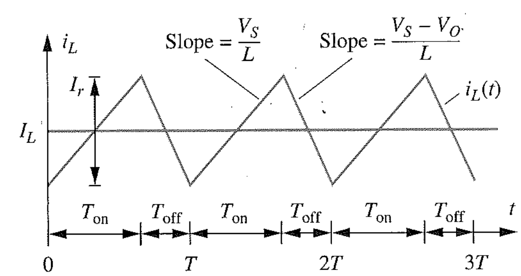


Figure 3.75 Periodic inductor current waveform: $i_L = I_L + i_r(t)$.

Switch S Open When the switch opens, the diode turns on, providing a path for the inductor current through the diode to load resistor R and filter capacitor C, as depicted in Fig. 3.74(b). To simplify the analysis, we assume that the ripple voltage at the output is small enough for the output voltage to be approximated by a dc voltage, $v_O \approx V_O$. Using this assumption, the voltage across the inductor is again constant and equal to $V_S - V_O$. The inductor current at the end of the T_{off} time interval $(t = T_{on} + T_{off} = T)$ is

$$i_L(T) = i_L(T_{on}) + \int_{T_{on}}^{T_{on}+T_{off}} \frac{V_S - V_O}{L}\, dt \tag{3.80}$$

$$i_L(T) = i_L(0^+) + \frac{V_S}{L} T_{on} + \frac{V_S - V_O}{L} T_{off} \tag{3.81}$$

For V_O exceeding V_S, the inductor current decreases with time during the T_{off} interval—again, as shown in Fig. 3.75. Also, because the circuit is operating periodically with period T, the inductor current at the times $t = 0^+$ and $t = T$ must be identical. Thus,

$$i_L(T) = i_L(0^+) \qquad \text{and} \qquad \frac{V_S}{L} T_{on} = \frac{V_O - V_S}{L} T_{off} \tag{3.82}$$

Rearranging this equation yields the basic relationship between the input and output voltage for the boost converter:

$$V_S(T_{\text{on}} + T_{\text{off}}) = V_O T_{\text{off}} \qquad \text{or} \qquad V_O = V_S \frac{T}{T_{\text{off}}} = \frac{V_S}{1 - \dfrac{T_{\text{on}}}{T}} = \frac{V_S}{1-\delta} \tag{3.83}$$

in which $\delta = T_{\text{on}}/T$ is termed the **duty cycle** of the switching waveform. The output voltage can be changed by varying the duty cycle of the switch. Because $0 \le \delta \le 1$, the output voltage $V_O \ge V_S$; the converter "boosts" the output voltage above the input voltage.

Inductor Design It is surprising to note that the expression for the output voltage in Eq. (3.83) is independent of L. An additional design parameter is needed in order to choose the value of inductor L. This parameter is the **ripple current** in the inductor. Because the voltage across the inductor is constant during both the T_{on} and T_{off} time intervals, the inductor current is a sawtooth waveform, as depicted in Fig. 3.75 [see Eqs. (3.79) and (3.80)]. The magnitude of the ripple current I_r is given by either

$$I_r = \frac{V_S}{L} T_{\text{on}} \qquad \text{or} \qquad I_r = \frac{V_O - V_S}{L} T_{\text{off}} \tag{3.84}$$

which must be equal. Rearranging Eq. (3.84) yields an expression for the value of the inductor:

$$L = \frac{V_S}{I_r} T_{\text{on}} = \frac{V_S T}{I_r}\left(\frac{T_{\text{on}}}{T}\right) = \frac{V_S}{I_r f}\delta \tag{3.85}$$

in which $f = 1/T$ is the switching frequency. From Eq. (3.85), we see that the higher the choice of operating frequency, the smaller the required value of inductance. dc-to-dc converters can be operated at frequencies well above 60 Hz in order to reduce the size of L, and f is normally chosen to be above the range of human hearing. Frequencies of 50 kHz to above 1 MHz are common.

dc Input Current In the boost circuit, the average inductor current I_L is larger than the dc load current. For the ideal converter, there is no loss mechanism within the circuit. Therefore, the power delivered to the input of the converter must equal the power delivered to load resistor R:

$$V_S I_S = V_O I_O \qquad \text{or} \qquad I_S = I_O \frac{V_O}{V_S} = I_O \frac{T}{T_{\text{off}}} = \frac{I_O}{1-\delta} \tag{3.86}$$

From this equation we see that the dc current in the inductor is greater than the dc load current by the same factor as the increase in output voltage. Note that the inductor must be properly designed to operate with this potentially large value of average current.

Ripple Voltage and Filter Capacitance In the boost converter, filter capacitor C is designed to control the ripple voltage V_r in a manner similar to that of the half-wave rectifier in Fig. 3.57 and Eq. (3.56). During time interval T_{on}, diode D is off, as in Fig. 3.74(a), and the capacitor must supply the total load current. If the ripple voltage is designed to be small, the discharge current is approximately constant and given by $I_O \cong V_O/R$. Based on this approximation, the ripple voltage can be expressed as

$$V_r \approx \frac{I_O}{C} T_{\text{on}} = \frac{V_O T_{\text{on}}}{RC} = \frac{V_O T}{RC}\left(\frac{T_{\text{on}}}{T}\right) = \frac{V_O T}{RC}\delta \tag{3.87}$$

Table 3.6 summarizes the design relationships for the dc-to-dc boost converter.

TABLE 3.6
Boost Converter Design

Output voltage	$V_O = V_S \dfrac{T}{T_{\text{off}}} = \dfrac{V_S}{1 - \dfrac{T_{\text{on}}}{T}} = \dfrac{V_S}{1-\delta}$
Source current	$I_S = I_O \left(\dfrac{T}{T_{\text{off}}}\right) = \dfrac{I_O}{1 - \dfrac{T_{\text{on}}}{T}} = \dfrac{I_O}{1-\delta}$
Inductor	$L = \dfrac{V_S}{I_r} T_{\text{on}} = \dfrac{V_S T}{I_r}\left(\dfrac{T_{\text{on}}}{T}\right) = \dfrac{V_S}{I_r f}\delta$
Filter capacitor	$C = \dfrac{V_O T}{V_r R}\left(\dfrac{T_{\text{on}}}{T}\right) = \dfrac{V_O T}{V_r R}\delta$

EXERCISE: **What is the duty cycle of the switching waveform in Fig. 3.73(b)?**

ANSWERS: **$\frac{2}{3}$ or 66.7%**

EXERCISE: **What are T, T_{on}, T_{off}, R, L, C, and I_L for a boost converter with the following specifications: $V_S = 5$ V, $V_O = 20$ V, $I_O = 1$ A, $I_r = 0.1$ A, $V_r = 0.25$ V, and $f = 50$ kHz? What are the values of v_L during the T_{on} and T_{off} time intervals?**

ANSWERS: **20 μs; 15 μs; 5 μs; 20 Ω; 0.75 mH; 60 μF; 4 A; +5 V; −15 V**

3.18.2 THE BUCK CONVERTER

The buck converter in Fig. 3.76 is designed to produce an output voltage that is smaller than the input voltage. Operation of the buck converter is similar to that of the boost converter, and switch S operates periodically with the same timing as in Fig. 3.73(b).

Switch S Closed During the time interval T_{on}, switch S is closed, and the positive input voltage reverse-biases diode D, resulting in the equivalent circuit in Fig. 3.76(b). We again assume that the ripple voltage at the output is small enough that the output voltage can be approximated by a constant voltage, $v_O \approx V_O$. Using this assumption, the voltage across the inductor is equal to $V_S - V_O$, and the inductor current at the end of the first T_{on} interval will be

$$i_L(T_{\text{on}}) = i_L(0^+) + \int_0^{T_{\text{on}}} \frac{V_S - V_O}{L}\, dt = i_L(0^+) + \frac{V_S - V_O}{L} T_{\text{on}} \tag{3.88}$$

Because the current in the inductor cannot change instantaneously, $i_L(0^+)$ is equal to the current just before the switch changes state.

Switch S Open When the switch opens, the diode turns on, providing a continuous path for the inductor current from ground through the diode to load resistor R and filter capacitor C, as depicted

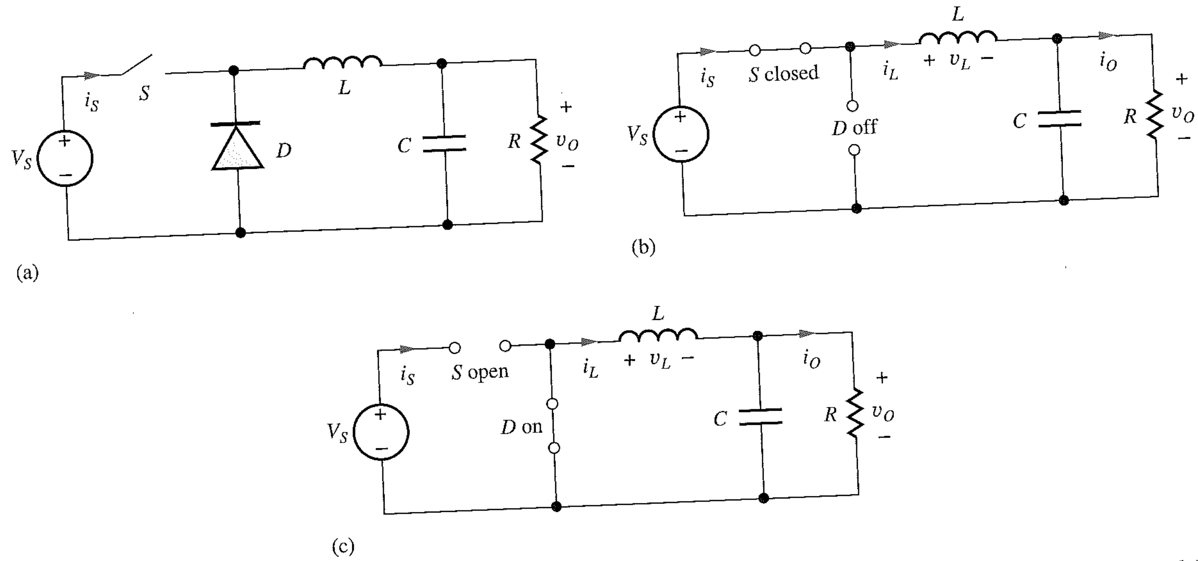


Figure 3.76 (a) dc-to-dc buck converter. (b) Model valid during T_{on} when switch S is closed. (c) Model for T_{off} interval when switch S is open.

in Fig. 3.76(c). The voltage across the inductor is now equal to $-V_O$. The inductor current at the end of the T_{off} time interval is

$$i_L(T) = i_L(T_{\text{on}}) + \int_{T_{\text{on}}}^{T_{\text{on}}+T_{\text{off}}} \frac{-V_O}{L}\,dt = i_L(0^+) + \frac{V_S - V_O}{L}T_{\text{on}} - \frac{V_O}{L}T_{\text{off}} \tag{3.89}$$

However, the circuit is again operating periodically with period T. Therefore, the inductor current at the times $t = 0^+$ and $t = T$ must be identical. Thus,

$$i_L(T) = i_L(0^+) \qquad \text{and} \qquad \frac{V_O - V_S}{L}T_{\text{on}} = \frac{V_O}{L}T_{\text{off}} \tag{3.90}$$

Rearranging this equation yields the basic relationship between the input and output voltage for the boost converter:

$$V_O = V_S\frac{T_{\text{on}}}{T} = V_S\delta \tag{3.91}$$

where δ is the switch duty cycle. Because $T_{\text{on}} \le T$, the output voltage $V_O \le V_S$. In this converter, the inductor voltage "bucks" the input voltage, and the converter output voltage is less than the input voltage. The output voltage of the buck converter is directly proportional to the duty cycle δ.

Inductor Design The relationship between the input and output voltages expressed by Eq. (3.91) is, again, independent of L, and the design of the inductor value is determined by the ripple current specification.

The inductor current waveform for the buck converter is very similar to that of the boost converter, as drawn in Fig. 3.77. The magnitude of the ripple current I_r is given by

$$I_r = \frac{V_S - V_O}{L}T_{\text{on}} = \frac{V_O}{L}T_{\text{off}} \tag{3.92}$$

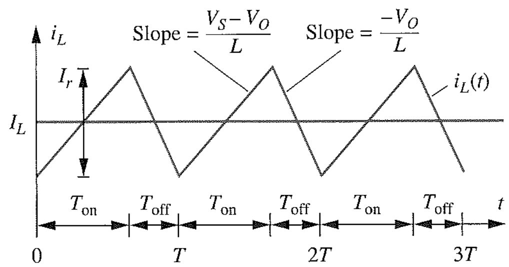


Figure 3.77 Inductor current waveform: $i_L = I_L + i_r(t)$.

Rearranging Eq. (3.91) yields an expression for the value of the inductor:

$$L = \frac{V_O}{I_r} T_{\text{off}} = \frac{V_O T}{I_r}\left(\frac{T_{\text{off}}}{T}\right) = \frac{V_O T}{I_r}\left(1 - \frac{T_{\text{on}}}{T}\right) = \frac{V_O}{I_r f}(1-\delta) \qquad (3.93)$$

In the buck converter, the dc current I_L is equal to the dc load current I_O. The current that must be supplied from the source V_S is given by

$$V_S I_S = V_O I_O \qquad \text{or} \qquad I_S = I_O \frac{V_O}{V_S} = I_O \frac{T_{\text{on}}}{T} = I_O\,\delta \qquad (3.94)$$

From this equation, we see that the dc input current to the converter is less than the load current.

Ripple Voltage and Filter Capacitance In the buck converter, only the ripple current must be absorbed by filter capacitor C. The positive change in voltage across the capacitor, which must also equal the negative voltage change, is equal to the ripple voltage V_r:

$$V_r = \frac{1}{C}\int_{T_{\text{on}}/2}^{T_{\text{on}}+(T_{\text{off}}/2)} i_r\,dt = \frac{\Delta Q}{C} \qquad \text{where } \Delta Q = \frac{1}{2}\left(\frac{I_r}{2}\right)\left(\frac{T_{\text{on}} + T_{\text{off}}}{2}\right) = \frac{I_r T}{8} \qquad (3.95)$$

The integral of the capacitor current represents the total change in charge ΔQ on the filter capacitor and corresponds to the area of the shaded triangular region in Fig. 3.77. Expressions for the value of the capacitor can be found using Eqs. (3.93) and (3.95):

$$C = \frac{I_r T}{8 V_r} = \frac{V_O}{V_r}\frac{T^2}{8L}(1-\delta) \qquad (3.96)$$

Table 3.7 summarizes the relationships needed for design of the buck converter.

TABLE 3.7
Buck Converter Design

Output voltage	$V_O = V_S \frac{T_{\text{on}}}{T} = V_S \delta$
Source current	$I_S = I_O \frac{T_{\text{on}}}{T} = I_O \delta$
Inductor	$L = \frac{V_O T}{I_r}\left(\frac{T_{\text{off}}}{T}\right) = \frac{V_O}{I_r f}(1-\delta)$
Filter capacitor	$C = \frac{I_r T}{8 V_r} = \frac{V_O}{V_r}\frac{T^2}{8L}(1-\delta)$

3.116. Draw a graph of the waveform for the current in the switch in Fig. 3.73 for the converter in the boost converter exercise (page 149). (b) Repeat for the current in diode D.

*3.117. (a) Derive an expression, similar to Eq. (3.83), for the output voltage of a boost converter using the CVD model for the diode. (b) What are the output voltage, ripple voltage, and ripple current of a boost converter operating with $V_S = 5$ V, $V_{on} = 0.75$ V, $R = 20\ \Omega$, $L = 0.75$ mH, $C = 60\ \mu$F, $f = 50$ kHz, and $\delta = \frac{3}{4}$? Compare your answer to the boost converter exercise in the text (page 149).

**3.118. The ideal boost converter has no loss (that is, its efficiency is 100 percent). (a) Derive an expression for the efficiency η of the boost converter, including the CVD model for the diode if $\eta = 100\% \cdot P_O/P_S = 100\% \cdot V_O I_O / V_S I_S$. (b) What is the efficiency of the boost converter in the exercise on page 149 if the diode voltage is 0.75 V? (c) Derive an expression for the efficiency η of the boost converter including both the on-voltage of the diode and a fixed voltage drop across the on switch.

Buck Converters

3.119. (a) Draw a graph of the waveform for the current in the source V_S in Fig. 3.76(a) for the converter in the buck converter exercise (page 152). (b) Repeat for the current in diode D.

3.120. Draw a graph, similar to Fig. 3.77, showing the waveforms of both the inductor current and output voltage for the buck converter in Fig. 3.76(a). Use the values from the buck converter exercise (page 152).

3.121. Design a buck converter operating at a frequency of 30 kHz to generate +15 V from a +50-V supply. The converter will have an output current of 0.5 A and a ripple voltage of less than 0.1 V. Assume that the ripple current is 10 percent of the dc inductor current.

3.122. Design a buck converter operating at a frequency of 50 kHz to generate +3.3 V from a +5 V supply. The converter will have an output current of 5 A, and a ripple voltage of less than 0.1 V. Assume that the ripple current is 10% of the dc inductor current.

3.123. Design a buck converter operating at a frequency of 50 kHz to generate +15 V from a +170-V supply. The converter will have an output current of 2.5 A and a ripple voltage of less than 0.5 V. Assume that the ripple current is 15 percent of the dc inductor current.

*3.124. (a) Derive an expression, similar to Eq. (3.91), for the output voltage of a buck converter using the CVD model for the diode. (b) What are the output voltage, ripple voltage, and ripple current of a buck converter operating with $V_S = 10$ V, $V_{on} = 0.75$ V, $R = 5\ \Omega$, $L = 1.25$ mH, $C = 1.25\ \mu$F, $f = 40$ kHz, and $\delta = \frac{1}{2}$? Compare your answer to the buck converter exercise (page 152).

**3.125. The ideal buck converter has no loss (that is, its efficiency is 100 percent). (a) Derive an expression for the efficiency η of the buck converter, including the CVD model for the diode if $\eta = 100\% \cdot P_O/P_S = 100\% \cdot V_O I_O / V_S I_S$. (b) What is the efficiency of the buck converter in the exercise on page 152 if the on-voltage of the diode is 0.75 V? (c) Derive an expression for the efficiency η of the buck converter, including both the on-voltage of the diode and a fixed voltage drop across the on switch.

3.19 Wave-Shaping Circuits

3.126. The circuit inside the box in Fig. 3.110 contains only resistors and diodes. The terminal V_O is connected to some point in the circuit inside the box. (a) Is the largest possible value of V_O most nearly 0 V, −6 V, +6 V, or +15 V? Why? (b) Is the smallest possible value of V_O most nearly 0 V, −9 V, +6 V, or +15 V? Why?

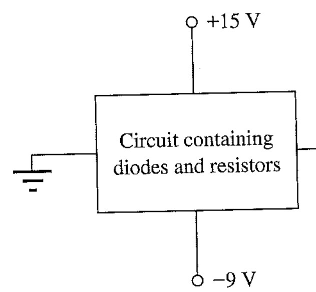


Figure 3.110

3.127. Draw waveforms for v_O for the two circuits in the Fig. 3.111. The diodes are ideal.

3.128. Draw the waveform for v_O for the circuit in Fig. 3.112. The diode has an on-voltage of 0.7 V and $R_Z = 0$. Use the waveform in Fig. 3.111.